

Arithmetic Sequences and Series

1. Express each of the following series in sigma notation.

(a) $1 + 2 + 3 + \dots + n$

(b) $1 + \frac{1}{4} + \frac{1}{9} + \dots + \frac{1}{n^2}$

(c) $\frac{1}{2} + \frac{4}{5} + \frac{3}{4} + \dots + \frac{2n+2}{n^2+4}$

(d) $-1 + 1 - 2 + \dots + (-1)^n(n-1)! + \dots$

(e) $\cos \theta + \cos^2 \theta + \dots + \cos^{88} \theta$

2. For each of the following arithmetic sequences:

i) Find the 64th term of the sequence.

ii) Find the sum of the first 14 terms of the sequence.

(a) $-8, -4, 0, 4, \dots$

(b) $62, 61, 60, 59, \dots$

(c) $7, 23, 39, 55, \dots$

(d) $1, \frac{3}{2}, 2, \frac{5}{2}, \dots$

(e) $-1, -7, -13, -19, \dots$

3. An arithmetic sequence has first term 2 and final last term 48.

Given that the sequence has twelve terms, find the sum of all terms in the sequence.

4. An arithmetic sequence has 11th term 30. The sum of the first 11 terms of the sequence is 55. What is the common difference for this sequence?

5. An arithmetic sequence has final term $l = 6$ and common difference $d = 3$

Given that the sum of all terms in the sequence is 0, find the last term of the sequence and the total number of terms in the sequence.

6. The sum of the first 32 terms of an arithmetic sequence is 1864. The sum of first 10 terms of the same sequence is 395.

(a) Find an expression for the n th term of this sequence.

(b) Find the 100th term of this sequence.

(c) After how many terms does the sum of the sequence exceed 2080?

7. A container is leaking at a constant rate of $\frac{3}{7}$ ml per minute. Given that the container is initially full of 6.4 litres of water, calculate how long it will take for the container to be empty.

8. Consider an arithmetic sequence with first term a and common difference d where $a, d \in \mathbb{R}$.

Let $S = a + (a + d) + (a + 2d) + \dots + (a + (n - 1)d)$.

Show that $S = \frac{n}{2}(2a + (n - 1)d)$.