The Exponential Function

1. Given $y = e^{2x}$:

(a) Express x in terms of y.

(b) Use your expression in (a) to find the value of x when $y = 2$ (to 4 s.f.).

2. Differentiate the following functions:

(a) $f(x) = e^x + 6x + 4$

(b) $g(x) = e^{x^2} - 6x^3$;

(c) $h(x) = 7x^3 + \sqrt{x} - 6e^{-x}$.

3. Find the stationary points of the following curves:

(a) $y = e^{-x^2} + x^2$

(b) $y = e^{(x^2+2x+1)}$

(c) $y = 2e^{(2x^3-x^2+1)}$

4. The decay in the number of radioactive nuclei in a certain substance is modelled by the curve $N = N_0e^{-kt}$ where $N_0 = 100,000$, $k = -0.1$. Here N is the number of radioactive nuclei left in the sample and t is the time in minutes since measurement of the decay began.

(a) How many radioactive nuclei were present in the sample when measurement began?

(b) How many radioactive nuclei will be left in the sample after 10 minutes to 3 s.f..

(c) How long, to the nearest minute, will it take for just half of the radioactive nuclei in the sample to remain?

(d) i. Find an expression for the rate of decay of radioactive nuclei in the substance
ii. Use your expression in i. to find the rate of decay of radioactive nuclei after 10 minutes to 3 s.f..

5. You are given the result that Given $\dfrac{d}{dx}(a^x) = a^x \ln a$ for real $a > 0$. Using this result find:

(a) $\dfrac{d}{dx}(2^x + 3^x)$.

(b) $\dfrac{d}{dx}((\sqrt{2})^x + (\sqrt{3})^x)$.

(c) The second derivative of $y = (\frac{1}{2})^x$.