

The Natural Logarithm

1. For each of the following equations, rearrange to make x the subject.

(a) $y = 3^{2x}$

(b) $p = e^{4x-8} + 6$

(c) $N = Ae^{-kx}$

(d) $q = 2^x \times 4^{2x}$

2. Find the following integrals:

(a) $\int \frac{1}{x} dx$

(b) $\int \left(x^2 + \frac{1}{x}\right) dx$

(c) $\int \left(2 + \frac{1}{x^2} - \frac{1}{x}\right) dx$

(d) $\int \left(e^x + \frac{2}{x}\right) dx.$

3. By using the fact that for a differentiable function $f(x)$

$$\int \frac{f'(x)}{f(x)} dx = \ln |f(x)| + C.$$

Find the following integrals:

(a) $\int \frac{2x + 2}{x^2 + 2x + 66} dx$

(b) $\int \frac{3x^2 + 6x}{x^3 + 3x^2} + 4 dx$

(c) $\int \frac{2e^{2x}}{e^{2x} - 4} dx$

(d) $\int \frac{x^3 + 6x^2 - 4}{x^4 + 8x^3 - 16x + 2} dx.$

4. Calculate the following definite integrals:

(a) $\int_1^2 \frac{1}{x} dx$

(b) $\int_3^4 \frac{1}{x^2} - \frac{1}{x} dx$

(c) $\int_2^4 \frac{3x^2 + 2x}{x^3 + x^2} dx$

(d) $\int_{-4}^{-1} \frac{x^5}{x^6 + 4} dx$

5. Find the area bounded by the curve $y = \frac{4x - 4}{x^2 - 2x + 3}$ and the lines $x = 1$ and $x = 9$.

6. Find the area bounded by the curve $y = \frac{\frac{1}{x}}{\ln x}$ and the lines $x = 3$ and $x = 4$.